REVIEW OF CHARM DALITZ PLOT ANALYSES

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For references given here in the form SMITH 05, see the references at the end of the D^+, D^0, and D_s^+ Listings.

The formalism of Dalitz-Plot analysis is reviewed in the preceeding note. Table 1 lists reported analyses of D mesons. In the following, we discuss a number of subjects of current interest: (1) $D^0 \to K_S^0\pi^+\pi^-$; (2) $D \to \pi\pi\pi$: a $\sigma(500)$ or $f_0(600)$; (3) $D^+ \to K^-\pi^+\pi^+$: a $\kappa(800)$? (4) the $f_0(980)$, $f_0(1370)$ and $f_0(1500)$; (5) doubly Cabibbo-suppressed decays; and (6) CP violation.

$\boldsymbol{D^0 \to K_S^0\pi^+\pi^-}$: Several experiments have analyzed $D^0 \to K_S^0\pi^+\pi^-$ decay (see Table 1). The most precise results are from CLEO (BABAR and Belle, discussed below, have not yet evaluated systematic uncertainties). The CLEO analysis included ten resonances: $K_S^0\rho^0$, $K_S^0\omega$, $K_S^0 f_0(980)$, $K_S^0 f_2(1270)$, $K_S^0 f_0(1370)$, $K^*(892)^-\pi^+$, $K_0^*(1430)^-\pi^+$, $K_2^*(1430)^-\pi^+$, $K^*(1680)^-\pi^+$, and the doubly Cabibbo-suppressed mode $K^*(892)^+\pi^-$. CLEO found a much smaller nonresonant contribution than did the earliest experiments.

The source of the nonresonant component found in the early experiments has been attributed to the broad scalar resonances, the $K_0^*(1430)^-$ and $f_0(1370)$, found in the later, larger data samples. The observation of a small but significant nonresonant component in the largest data samples suggests the presence of additional broad scalar resonances, the $\kappa(800)$ and $\sigma(500)$. The CLEO analysis could accommodate the $\sigma(500)$ in lieu of the nonresonant component, but found no evidence for the $\kappa(800)$.

The ten quasi-two-body intermediate states in the CLEO analysis include both CP-even and CP-odd eigenstates and one doubly Cabibbo-suppressed channel. A time-dependent analysis of the Dalitz plot allows simultaneous determination of the strong transition amplitudes and phases and the mixing parameters x and y without phase or sign ambiguity. Using 9 fb^{-1}, CLEO obtained $(-4.5 < x < 9.3)\%$ and $(-6.4 < y < 3.6)\%$ [1].

The decay $D^0 \to K^0_S\pi^+\pi^-$, important for the study of the CKM angle γ/ϕ_3 [6], is under study by Belle [10,11] and BABAR [12,13]. The CLEO model does not provide a good description of the higher-statistics BABAR and Belle data samples. An improved description is obtained in two ways: First, by adding more Breit-Wigner resonances, including two $\pi\pi$ resonances with arbitrary mass and width, denoted as σ_1

Table 1: Reported Dalitz plot analyses.

Decay	Experiment(s)
$D^0 \to K^0_S\pi^+\pi^-$	Mark II [a], Mark III [b], E691 [c], E687 [d,e], ARGUS [f], CLEO [g], Belle [10,11], BABAR [12,13]
$D^0 \to K^-\pi^+\pi^0$	Mark III [b], E687 [e], E691 [c], CLEO [h]
$D^0 \to \overline{K}^0K^+\pi^-$	BABAR [14]
$D^0 \to K^0K^-\pi^+$	BABAR [14]
$D^0 \to K^0_S\eta\pi^0$	CLEO [i]
$D^0 \to \pi^+\pi^-\pi^0$	CLEO [j]
$D^0 \to K^0_SK^+K^-$	BABAR [k]
$D^0 \to K^-K^+K^-\pi^+$	FOCUS [l]
$D^0 \to K^-K^+\pi^-\pi^+$	FOCUS [m]
$D^+ \to K^-\pi^+\pi^+$	Mark III [b], E687 [e], E691 [c], E791 [n]
$D^+ \to \overline{K}^0\pi^+\pi^0$	Mark III [b]
$D^+ \to \pi^+\pi^+\pi^-$	E687 [o], E791 [p], FOCUS [5] [q]
$D^+ \to K^+K^-\pi^+$	FOCUS [15], E687 [r], BABAR [s]
$D^+ \to K^+\pi^+\pi^-$	E791 [t], FOCUS [u]
$D^+_s \to K^+K^-\pi^+$	E687 [r], FOCUS [15]
$D^+_s \to \pi^+\pi^+\pi^-$	E687 [o], E791 [v], FOCUS [5]
$D^+_s \to K^+\pi^+\pi^-$	FOCUS [u]

See the end of the D^+, D^0 and D^+_s Listings for these references: [a]SCHINDLER 81, [b]ADLER 87, [c]ANJOS 93, [d]FRABETTI 92B, [e]FRABETTI 94G, [f]ALBRECHT 93D, [g]MURAMATSU 02, [h]KOPP 01, [i]RUBIN 04, [j]CRONIN-HENNESSY 05, [k]AUBERT 05B, [l]LINK 03G, [m]LINK 05C, [n]AITALA 02, [o]FRABETTI 97D, [p]AITALA 01B, [q]LINK 04, [r]FRABETTI 95B, [s]AUBERT 05A, [t]AITALA 97C, [u]LINK 04F, [v]AITALA 01A.

and σ_2. Second, following the methodology of FOCUS [LINK 04], by applying a K-matrix model to the $\pi\pi$ S-wave [12].

Charm Dalitz-plot analyses might also prove useful for calibrating tools used to study B decays: specifically, to extract α from $B^0 \to \pi^+\pi^-\pi^0$, β from $b \to s$ penguin decays (*e.g.*, $B^0 \to \overline{K}^0 K^+K^-$), and γ from $B^\pm \to DK^\pm$ followed by $D^0 \to \pi^+\pi^-\pi^0$ or $K^0_S K^+K^-$ or $K^+K^-\pi^0$, in addition to the well-studied $D^0 \to K^0_S\pi^+\pi^-$[2, 3].

$D \to \pi\pi\pi$: a $\sigma(500)$ or $f_0(600)$: The decay $D^+ \to \pi^+\pi^+\pi^-$ has been studied by the E687, E791 and FOCUS experiments (see Table 1). The E687 analysis considered the modes $\rho(770)^0\pi^+$, $f_0(980)\pi^+$, $f_2(1270)\pi^+$, and a nonresonant component. E791 included, in addition, $f_0(1370)\pi^+$ and $\rho(1450)^0\pi^+$. Both analyses found a very large fraction ($\sim$ 50%) for the nonresonant component, perhaps indicating a broad scalar contribution. E791 found the nonresonant amplitude to be consistent with zero if a broad scalar resonance was included. FOCUS analyzed its data using both the Breit-Wigner formalism and the K-matrix formalism for the $\pi^+\pi^-$ S-wave, following a 5-pole, 5-resonance model of Anisovich and Sarantsev [16]. The Breit-Wigner analysis included $\rho(770)^0, f_0(980), f_2(1270)^0, f_0(1500), \sigma(500)$, and a nonresonant component. The K-matrix formalism, with Breit-Wigner forms for the $\rho(770)$ and $f_2(1270)$, also describe the FOCUS data well. None of these analyses has modeled the dynamics of the $\pi^+\pi^+$ interaction. Consideration of the $I = 2$ S- and D-wave phase shifts, also measured in $\pi^+p \to \pi^+\pi^+n$ [18], could affect the $\pi^+\pi^-$ S-wave result.

Using the E791 data, Bediaga and Miranda [19] found additional evidence that the low-mass $\pi^+\pi^-$ feature is resonant by examining the phase of the $\pi^+\pi^-$ amplitude in the vicinity of the reported $\sigma(500)$ mass. The phase variation with invariant mass is consistent with a resonant interpretation.

Table 1 gives the parameters of the $\sigma(500)$ determined in charm Dalitz-plot analyses. A consistent relative phase between the $\sigma(500)$ and $\rho(770)$ resonances is observed.

Table 2: Parameters of the $\sigma(500)$ resonance. The amplitude and phase are relative to the $\rho(770)$.

Experiment	E791[a]	CLEO[b]	FOCUS [5]
Decay mode	$D^+ \to \pi^+\pi^+\pi^-$	$D^0 \to K_S^0\pi^+\pi^-$	$D^+ \to \pi^+\pi^+\pi^-$
Amplitude	$1.17 \pm 0.13 \pm 0.06$	0.57 ± 0.13	—
Phase (°)	$205.7 \pm 8.0 \pm 5.2$	214 ± 11	200 ± 31
m (MeV/c^2)	$478^{+24}_{-23} \pm 17$	513 ± 32	443 ± 27
Γ (MeV/c^2)	$324^{+42}_{-40} \pm 21$	335 ± 67	443 ± 80

See the end of the D^+ and D^0 listings for these references: [a]AITALA 01B, [b]MURAMATSU 02.

CLEO has studied $D^0 \to \pi^+\pi^-\pi^0$ (see Table 1). Only the three $\rho(770)\pi$ resonant contributions are observed. No evidence is found for any $\pi\pi$ S-wave, either with the Breit-Wigner or with a K-matrix parametrization, using the 4-pole, 2-resonance model of Au, Morgan, and Pennington [17].

$D^+ \to K^-\pi^+\pi^+$: a $\kappa(800)$?: Evidence for a broad $K\pi$ scalar resonance has been found by E791 in $D^+ \to K^-\pi^+\pi^+$ (see Table 1). Fitting the Dalitz plot with $\overline{K}^*(892)^0\pi^+$, $\overline{K}_0^*(1430)^0\pi^+$, $\overline{K}_2^*(1430)^0\pi^+$, and $\overline{K}^*(1680)^0\pi^+$, plus a constant nonresonant component, E791 found results consistent with earlier analyses by E691 and E687, with a nonresonant fit fraction of over 90%. With more events than the other experiments, E791 was then led to include an extra low-mass S-wave $\overline{K}\pi$ resonance to account for the poor fit already seen by earlier experiments. A $\kappa(800)$ with mass $797 \pm 19 \pm 43$ MeV and width $410 \pm 43 \pm 87$ MeV much improved the fit. The $\kappa(800)$ became the dominant resonance and the nonresonant fit fraction was reduced from $90.9 \pm 2.6\%$ to $13.0 \pm 5.8 \pm 4.4\%$.

In addition, E791 modeled the $K\pi$ S-wave phase variation as a function of $K\pi$ mass with only the $K_0^*(1430)$ resonance and a nonresonant component following a parametrization of LASS [20]. It was necessary to relax the unitarity constraint to describe the data [21]. The $K\pi$ S-wave phase behavior in this model was consistent with the model that included the κ resonance.

Finally, E791 performed a model-independent partial-wave analysis [AITALA 05] of the S-wave component of the $K\pi$ system, finding the amplitude and phase from the $K\pi$ threshold up to 1.72 GeV. No assumptions were made regarding dependence on invariant mass, but the analysis did use the relatively well-understood P- and D-waves, described by the $K^*(892)$ and $K^*(1680)$ and by the $K_2(1430)$, respectively. The results were similar to those obtained by AITALA 02, which parametrized the S-wave with κ and $K_0(1430)$ Breit-Wigner forms and a constant complex non-resonant term. As with the $\sigma(500)$, the $K^-\pi^+$ S-wave result could be affected by including dynamics of the $I = 2$ $\pi^+\pi^+$ interaction; however in AITALA 05, the $I = 2$ elastic amplitude was found to be negligible compared to the κ.

CLEO allowed scalar $K\pi$ resonances in fits to $D^0 \to K^-\pi^+\pi^0$ and $D^0 \to K^0_S\pi^+\pi^-$ (see Table 1), and observed a significant contribution from only the $K^*_0(1430)$ [22]. BABAR fit $D^0 \to K^0K^-\pi^+$ with both positively charged and neutral $\overline{K}^*(892)$, $\overline{K}^*_0(1430)$, $\overline{K}^*_2(1430)$, and $\overline{K}^*(1680)$ resonances, as well as the $a_0(980)^-$, $a_0(1450)^-$, and $a_2(1310)^-$ resonances, and a nonresonant component [14]. BABAR also fit $D^0 \to \overline{K}^0K^+\pi^-$ with the same resonances except for the $a_2(1310)^-$. In both cases, a good fit was obtained without including the κ.

FOCUS has conclusively observed a $K\pi$ S-wave as a distortion of the $K^*(892)$ line-shape in semileptonic charm decays [LINK 02E, LINK 05D].

The $f_0(980)$, $f_0(1370)$ and $f_0(1500)$: The meson content of the 0^{++} nonet and the quark content of the $f_0(980)$, $a_0(980)$, $f_0(1370)$, $f_0(1500)$, and $f_0(1710)$ mesons are current puzzles in light-meson spectroscopy [22]. Measuring branching fractions and couplings to different final states and comparing scalar-meson production rates among D^0, D^+, and D_s^+ mesons may help solve these puzzles.

For example: A large contribution of $f_0(980)$ to $D^0 \to K^0_SK^+K^-$ was reported by ARGUS [ALBRECHT 87E] and by BABAR [14]. This is inconsistent with the smaller contribution of $f_0(980)$ observed in $D^0 \to K^0_S\pi^+\pi^-$ by CLEO. The explanation is that $D^0 \to K^0_SK^+K^-$ has a large contribution from $a_0(980)^0 \to K^+K^-$. Therefore CLEO studied

$D^0 \to K^0_S \eta\pi^0$ [RUBIN 04], where the dominant contribution is from $K^0_S\, a_0(980)^0, a_0(980)^0 \to \eta\pi^0$, and there can be no $f_0(980)$. A more recent BABAR analysis of $D^0 \to K^0_s K^+ K^-$ found a large amount of $a_0(980) \to K\overline{K}$ and little $f_0(980)$ [AUBERT 05B].

The proximity of the $K\overline{K}$ threshold requires either a coupled-channel Breit-Wigner function [23] or a Flatte parametrization [24] of the $f_0(980)$. The width of the $f_0(980)$ is poorly known. E791 and FOCUS [LINK 05C] [5] used a coupled-channel Breit-Wigner function to describe the $f_0(980)$ in $D^+_s \to \pi^+\pi^+\pi^-$. BESII studied the $f_0(980)$ in $J/\psi \to \phi\pi^+\pi^-$ and ϕK^+K^- [25]. The values found for the couplings to the $\pi\pi$ and $K\overline{K}$ channels, $g_{\pi\pi}$ and g_{KK}, were not consistent. Results such as these are desirable for input to the analysis of $D^+_s \to K^+K^-\pi^+$ [15], which includes both the $f_0(980)$ and $a_0(980)$.

The quark content of the $f_0(1370)$ and $f_0(1500)$ can perhaps be inferred from how they populate various Dalitz plots. Results so far are confusing. The E791 analysis of $D^+ \to \pi^+\pi^+\pi^-$ [AITALA 01B] found some $f_0(1370)$ but no $f_0(1500)$, while the FOCUS analysis [5] of this mode found little $f_0(1370)$. In $D^+_s \to \pi^+\pi^+\pi^-$, E687 and FOCUS [5] found no $f_0(1370)$, but did find a resonance with parameters similar to the $f_0(1500)$, whereas E791 found a $\pi^+\pi^-$ resonance with mass $1434 \pm 18 \pm 9$ MeV and width $172 \pm 32 \pm 6$ MeV, consistent with neither the $f_0(1370)$ or $f_0(1500)$. BABAR [AUBERT 05B] in $D^0 \to \overline{K}^0 K^+K^-$ found neither the $f_0(1370)$ nor the $f_0(1500)$, but did observe a K^+K^- resonance consistent with the values from E791 given above, while CLEO has observed the $f_0(1370)$ in $D^0 \to K^0_S\pi^+\pi^-$. The FOCUS analysis that used the K-matrix formalism for the $\pi\pi$ S-wave observed significant couplings to five T-matrix poles—$f_0(980), f_0(1300), f_0(1200-1600), f_0(1500), f_0(1750)$— in both $D^+ \to \pi^+\pi^-\pi^+$ and $D^+_s \to \pi^+\pi^-\pi^+$. Again, the quark content of each pole might be inferred from the coupling to various Dalitz plots.

It is noteworthy that the S-wave observed in B Dalitz-plot analyses appears to be different than that observed in D-meson decays.

Doubly Cabibbo-Suppressed Decays: There are two classes of multibody doubly Cabibbo-suppressed (DCS) decays of D mesons. The first consists of those in which the DCS and corresponding Cabbibo-favored (CF) decays populate distinct Dalitz plots: the pairs $D^0 \to K^+\pi^-\pi^0$ and $D^0 \to K^-\pi^+\pi^0$, or $D^+ \to K^+\pi^+\pi^-$ and $D^+ \to K^-\pi^+\pi^+$, are examples. CLEO [BRANDENBURG 01] and Belle [TIAN 05] have reported $\Gamma(D^0 \to K^+\pi^-\pi^0)/\Gamma(D^0 \to K^-\pi^+\pi^0) = (0.43^{+0.11}_{-0.10} \pm 0.07)\%$ and $(0.229 \pm 0.015^{+0.013}_{-0.009})\%$, respectively. E791 and FOCUS have reported $\Gamma(D^+ \to K^+\pi^-\pi^+)/\Gamma(D^+ \to K^-\pi^+\pi^+) = (0.77 \pm 0.17 \pm 0.08)\%$ and $(0.65 \pm 0.08 \pm 0.04)\%$, respectively.

The second class consists of decays in which the DCS and CF modes populate the same Dalitz plot; for example, $D^0 \to K^{*-}\pi^+$ and $D^0 \to K^{*+}\pi^-$ both contribute to $D^0 \to K^0_S\pi^+\pi^-$. In this case, the potential for interference of DCS and CF amplitudes increases the sensitivity to the DCS amplitude. CLEO found the relative amplitude and phase to be $(7.1 \pm 1.3^{+2.6}_{-0.6}{}^{+2.6}_{-0.6})\%$ and $(189 \pm 10 \pm 3^{+15}_{-5})^\circ$, corresponding to $\Gamma(D^0 \to K^*(892)^+\pi^-)/\Gamma(D^0 \to K^*(892)^-\pi^+) = (0.5 \pm 0.2^{+0.5}_{-0.1}{}^{+0.4}_{-0.1})\%$. In addition to $D^0 \to K^*(892)^+\pi^-$, Belle [10,11] and

BABAR [12,13] have found evidence for $D^0 \to K_0(1430)^+\pi^-$ and $K_2(1430)^+\pi^-$, and Belle has also found evidence for $K^*(1680)^+\pi^-$.

CP Violation: In the limit of CP conservation, charge conjugate decays will have the same Dalitz-plot distribution. The $D^{*\pm}$ tag enables the discrimination between D^0 and $\overline{D}^0$. The integrated CP violation across the Dalitz plot is determined in two ways. The first uses

$$\mathcal{A}_{CP} = \int \left(\frac{|\mathcal{M}|^2 - |\overline{\mathcal{M}}|^2}{|\mathcal{M}|^2 + |\overline{\mathcal{M}}|^2} \right) dm^2_{ab}\, dm^2_{bc} \ \Big/ \int dm^2_{ab}\, dm^2_{bc}\,, \quad (1)$$

where $\mathcal{M}$ and $\overline{\mathcal{M}}$ are the D^0 and $\overline{D}^0$ Dalitz-plot amplitudes. The second uses the asymmetry in the efficiency-corrected D^0 and $\overline{D}^0$ yields,

$$\mathcal{A}_{CP} = \frac{N_{D^0} - N_{\overline{D}^0}}{N_{D^0} + N_{\overline{D}^0}} . \qquad (2)$$

These expressions are less sensitive to CP violation than are the individual resonant submodes [ASNER 04A]. Table 3 lists the results for CP violation. No evidence of CP violation has been observed in D-meson decays.

Table 3: Dalitz-plot-integrated CP violation. Measurements computing $\mathcal{A}_{CP}$ with Eq. (2) rather than Eq. (1) are denoted †.

Experiment	Decay mode	$\mathcal{A}_{CP}(\%)$
BABAR [a]	$D^+ \to K^+K^-\pi^+$	$1.4 \pm 1.0 \pm 0.8$
Belle [b†]	$D^0 \to K^+\pi^-\pi^0$	-0.6 ± 5.3
Belle [b†]	$D^0 \to K^+\pi^-\pi^+\pi^-$	-1.8 ± 4.4
CLEO [c]	$D^0 \to K^-\pi^+\pi^0$	-3.1 ± 8.6
CLEO [d†]	$D^0 \to K^+\pi^-\pi^0$	$+9^{+22}_{-25}$
CLEO [e]	$D^0 \to K^0_S\pi^+\pi^-$	$-0.9 \pm 2.1^{+1.0+1.3}_{-4.3-3.7}$
CLEO [f]	$D^0 \to \pi^+\pi^-\pi^0$	$+1^{+9}_{-7} \pm 9$

See the end of the D^+ and D^0 Listings for these references: [a]AUBERT 05A, [b]TIAN 05, [c]KOPP 01, [d]BRANDENBURG 01, [e]ASNER 04A, [f]CRONIN-HENNESSY 05.

The possibility of interference between CP–conserving and CP–violating amplitudes provides a more sensitive probe of CP violation. The constraints on the square of the CP–violating amplitude obtained in the resonant submodes of $D^0 \to K^0_S\pi^+\pi^-$ range from 3.5×10^{-4} to 28.4×10^{-4} at 95% confidence level [ASNER 04A].

References

1. See the note on "D^0–$\overline{D}^0$ Mixing" in this *Review*.
2. See the note on "The CKM Quark Mixing Matrix" in this *Review*.
3. See the note on "CP Violation in Meson Decays" in this *Review*.

4. Dalitz plot analysis of the wrong sign rate $D^0 \to K^+\pi^-\pi^0$ [BRANDENBURG 01] and the time dependence of Dalitz plot analysis of $D^0 \to K^0_S\pi^+\pi^-$ [ASNER 05] are two candidate processes.
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